Phys 8501 Lecture 1
General Relativity

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Text. "An Intro to GR Spacetime & Geometry" by Sean Carroll
Also "General Relativity" R. Wald
"Gravitation & Cosmology" S. Weinberg

Outline (see Carroll's book too)

Introduction; Special Relativity	1 week
Manifolds	2 wks
Parallel transport, Curvature	3 wks
Gravity, GR	3 wks
Black holes, the Schwarzschild solution	2 wks
More general black holes; Kerr solution	2 wks
Gravitational radiation	2 wks

General relativity

A theory of the structure of space and time ie space and time relationships between events; also a theory of gravity!

Remarkably the math tools needed to describe spacetime structure in GR is essentially the same as that needed in ordinary geometry (of curved spaces)

Basic question

Given clocks and meter sticks determine space and time relationships between events.

→ characterize fundamental intrinsic structure of spacetime ≡ set of events.

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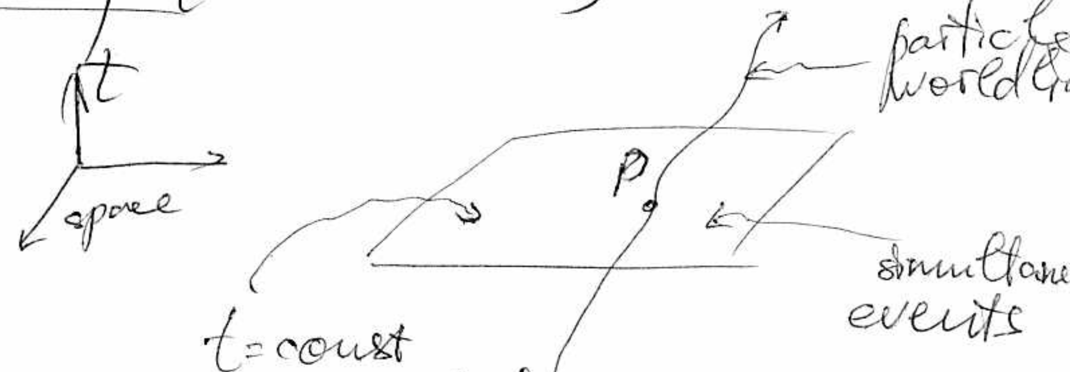
Space and time notions

Pre Relativity (Newtonian)

Causual structure

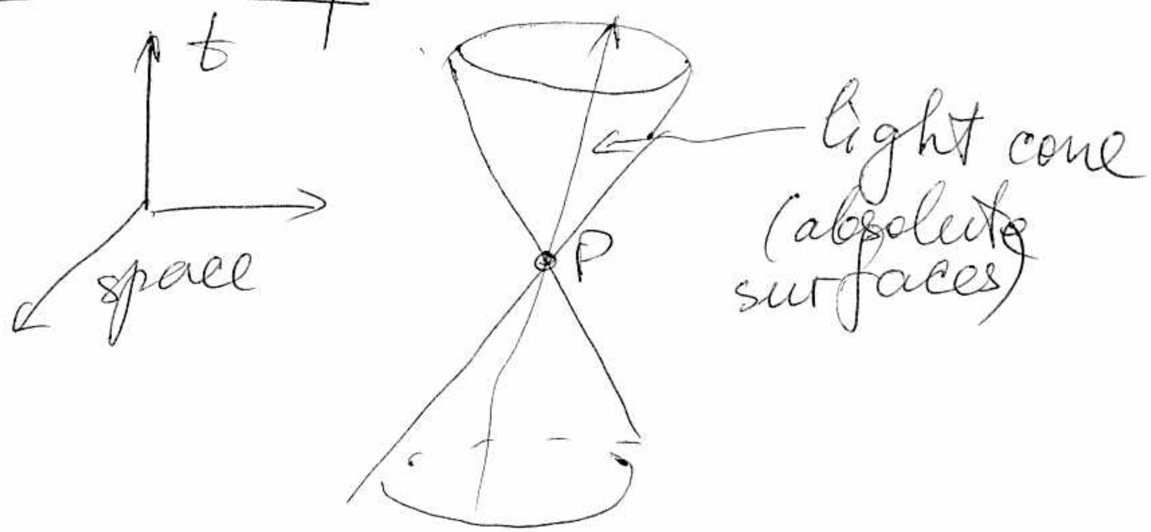


- There is an absolute slicing in spacetime of 3d surfaces defined by $t = \text{const}$
  $\longrightarrow$ absolute simultaneity
- Any curve in spacetime which is non-tangential to $t = \text{const}$ surface is a possible motion for a particle.
- Invariant spacetime structure
  (i) Δt
  (ii) spatial (interval)^2 $(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$
  between simultaneous events

This picture is NOT correct ($\Delta t = 0$)

Special relativity

Causual structure



Замечаем делая опыты $\rightarrow \infty$ или очень велика скор. света

- No absolute simultaneity
  i.e. there is no well definede notion of where in spacetime $t = \text{const}$ surfaces lie.

Define inertia frame or coordinates
Motion of grid is unaccelerated.
Synchronize each pair of clocks by sending signals back and forth in a symmetrical way.

- Particles always travel at less than or equal to the speed of light

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- Invariant structure

spacetime interval $\boxed{(\Delta s)^2 = -(c\Delta t)^2 + (\Delta s^2 + \Delta y)^2}$

Invariant under changes of inertial coords.

Minkowski's space

= a four-dimensional spacetime. labelled by coordinates

x^{μ} Greek superscript $(x^0, x^1, x^2, x^3) = (ct, \underbrace{x, y, z}_{x^i})$

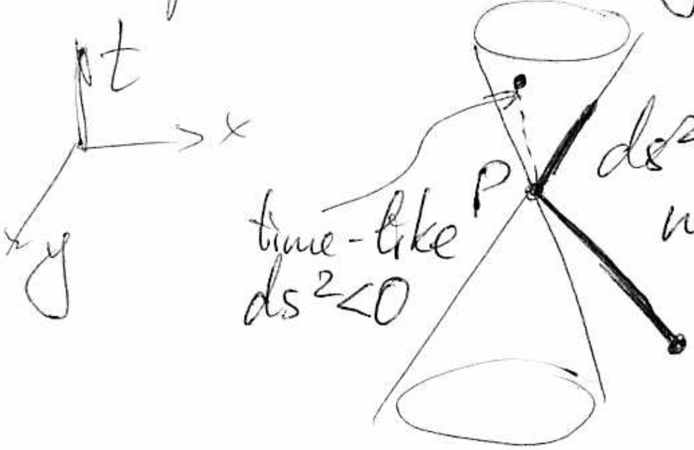
Define metric $\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Then for infinitesimal displacement dx^{μ}

$$\boxed{ds^2 = \eta_{\mu\nu} dx^{\mu} dx^{\nu}}$$

Note: summation convention over repeated upper and lower indices.

Spacetime diagram



Proper-time $(d\tau)^2 = -ds^2 =$

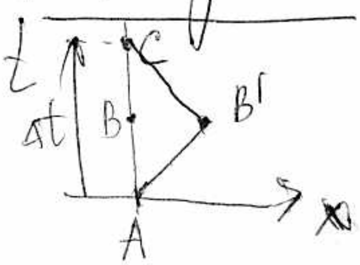
$$= -\eta_{\mu\nu}dx^\mu dx^\nu$$

→ measures the time elapsed between two events as seen by an observer moving on the straight path between the events

Note! : The proper time τ and coordinate time t are different

eg: Twin paradox $\Delta\tau_{AB'C} = \sqrt{1-v^2}\,\Delta t$

$$< \Delta t = \Delta\tau_{ABC}$$



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Nonstraight path has the shorter proper time.

Lorentz transformations

What are the transformations that leave $(\Delta s)^2$ invariant?

Translations $x^\mu \to x^{\mu'} = \delta_\mu^{\mu'}(x^\mu + a^\mu)$

$$\delta_\mu^{\mu'} = \begin{cases} 1 & \mu' = \mu \\ 0 & \mu' \neq \mu \end{cases}, \quad \text{where } a^\mu = \text{const.}$$

$\Rightarrow \Delta x = 0.$

Spatial rotations & boosts

$$x^{\mu'} = \underbrace{\Lambda^{\mu'}_{\nu}}_{4\times 4 \text{ matrix}} x^\nu$$

What are the matrices Λ?