Phys 8501 Lecture 4 (M 09.20.04)

Need to modify SR
— does not include gravity
Einstein was motivated by <u>Principle of Equivalence</u>

Newton's 2nd law $F = \underbrace{m_i}_{\text{inertial mass}} a$

Newton's law of gravitation $F_g = m_g a_g$

Objects fall at the same rate $\Rightarrow m_i = m_g$
Known as <u>Weak Equivalence Principle</u>

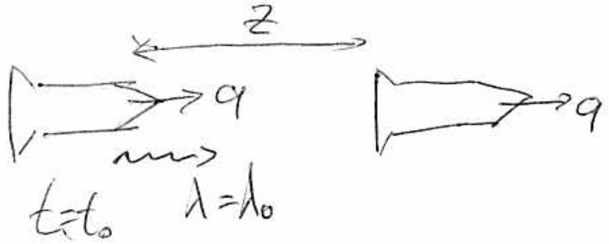
<u>Einstein EP</u> (strong EP "gravitational")
In small enough regions of spacetime the (nongravitational) laws of physics reduce to SR;
It is impossible to detect the existence of gravitational field by means of local experiments.

$\Rightarrow$ acceleration due to gravity ~~i.e.~~
i.e. gravity is <u>not</u> force.

$\Rightarrow$ define "unaccelerated" to be inertial

Due to inhomogeneities in gravitational field only have <u>locally</u> inertial frames
$\rightarrow$ No globa reference frame

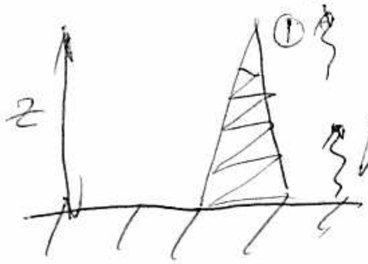
Gravitational redshift



The Doppler effect:

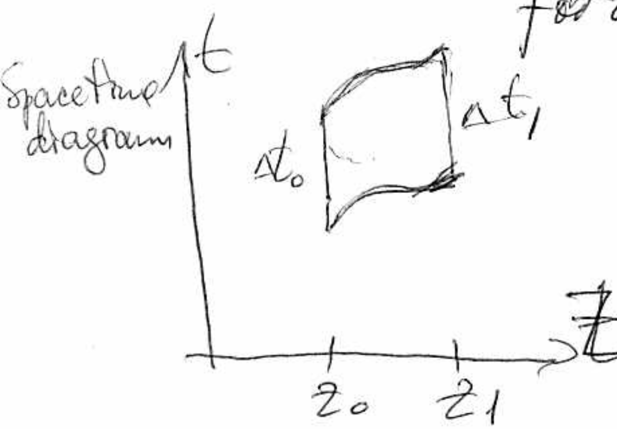
$$\frac{\Delta\lambda}{\lambda} = \frac{az}{c^2}$$

But EEP $\Longrightarrow$ Same thing ~~in~~ Gravitational field



$$\frac{\Delta\lambda}{\lambda_0} = \frac{a_g z}{c^2}$$

or $\underbrace{\Delta t_1}_{\text{time interval for absorption}} > \underbrace{\Delta t_0}_{\text{time interval for emission}}$



Flat geometry $\Rightarrow \Delta t_0 = \Delta t_1$!
× not correct

<u>Conclusion</u> Spacetime must have a mathematical structure that ~~looks~~ like Minkowski space but has a curved geometry ($\Rightarrow$ gravity)

$\Longrightarrow$ manifold

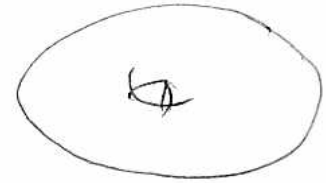
<u>Manifolds</u>

A n-dim set which <u>locally</u> "looks like" $\mathbb{R}^n$ in continuity and smoothness.

<u>Examples</u>

S^2

torus $S^1 \times S^1$



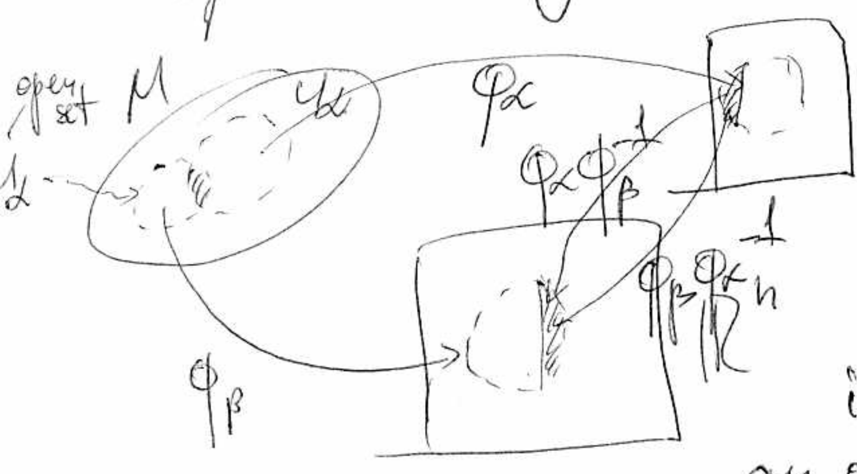
<u>Not</u> a manyfold

A function is C^p if its <u>pth</u> derivative exists and is continuous

C^∞ maps are <u>smooth</u>.

<u>n-dim manifold</u> M

Set obtainable by "smoothly sewing together" open ball $|x-y|<R$.

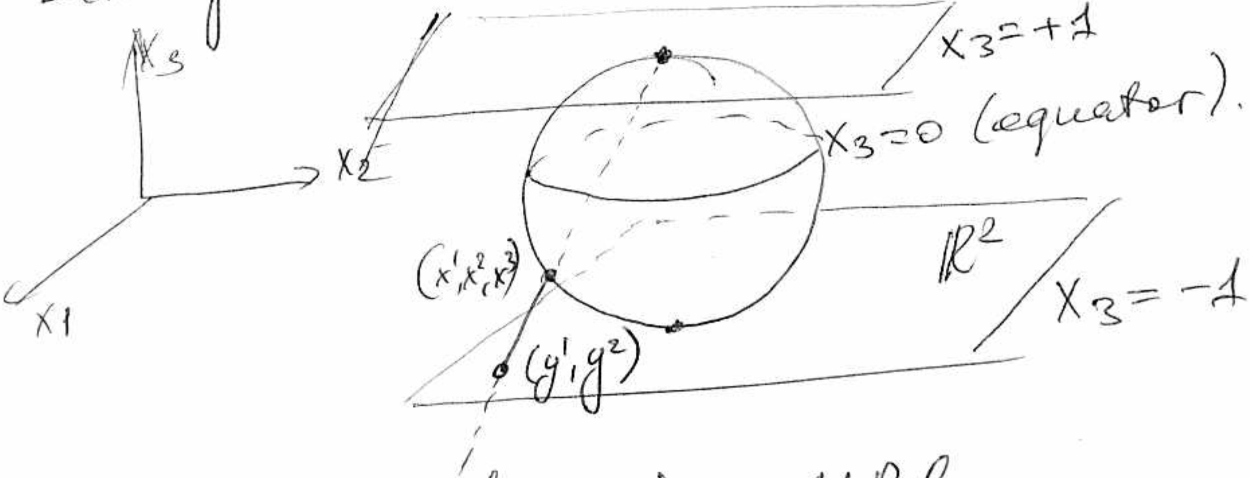


$\{(U_\alpha, \varphi_\alpha)\}$ is an <u>atlas</u>
<u>chart</u>

Require each shaded region i.e. each $\varphi_\alpha[U_\alpha \cap U_\beta]$ to be an open subset of $\mathbb{R}^n$ and reque $\varphi_\alpha \circ \varphi_\beta^{-1}$ is <u>smooth</u> (C^∞)

Two sets M and N are diffeomorphic if there exists a C^∞ map $\phi: M \to N$ with C^∞ inverse $\phi^{-1}: N \to M$; ϕ is a <u>diffeomorphism</u>

Example: S^2 $\quad (x^1)^2 + (x^2)^2 + (x^3)^2 = 1$



U_1 = sphere minus N Pole

$$\phi_1(x^1 x^2 x^3) \equiv (y^1, y^2) = \left(\frac{2x^1}{1-x^3}, \frac{2x^2}{1-x^3}\right)$$

$(U_1 \phi_1)$ form a chart

Can form another chart $(U_2 \phi_2)$

$$\phi_2(x^1 x^2 x^3) = (z^1, z^2) = \left(\frac{2x^1}{1+x^3}, \frac{2x^2}{1+x^3}\right)$$

U_2 = sphere minus S Pole

$\{(U_1 \phi_1)(U_2 \phi_2)\}$ atlas

<u>Check that</u> $\phi_2 \circ \phi_1^{-1}$ ~~is~~ given by $\quad -1 < x_3 < 1$

$$z^i = \frac{4y^i}{(y^1)^2 + (y^2)^2}$$

<u>is C^∞</u>