

Need to modify SR

— does not include gravity

Einstein was motivated by Principle of Equivalence

Newton's 2nd law $F = m_i a$
inertial mass

Newton's law of gravitation $F_g = m_g a_g$

Objects fall at the same rate $\Rightarrow m_i = m_g$
Known as Weak Equivalence Principle

Einstein EP (strong EP "gravitational")

In small enough regions of spacetime the (nongravitational) laws of physics reduce to SR;
It is impossible to detect the existence of gravitational field by means of local experiments.

\Rightarrow acceleration due to gravity $\neq 0$.

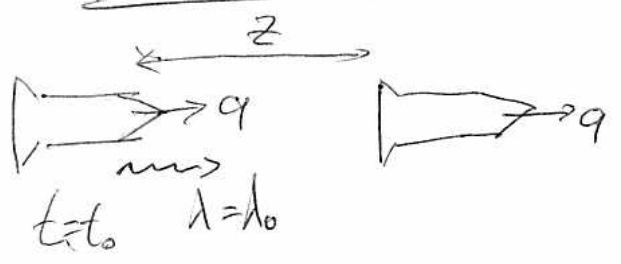
i.e. gravity is not force.

\Rightarrow define "unaccelerated" to be inertial

Due to inhomogeneities in gravitational field only have locally inertial frames

\rightarrow No global reference frame

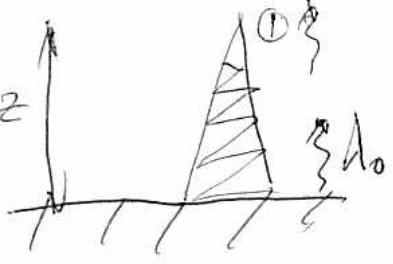
Gravitational redshift



The Doppler effect:

$$\frac{\Delta \lambda}{\lambda} = \frac{a \Delta z}{c^2}$$

But EEP \Rightarrow Same thing in gravitational field

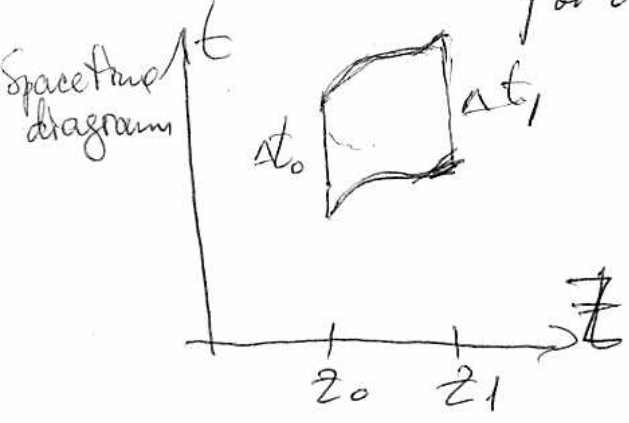


$$\frac{\Delta \lambda}{\lambda_0} = \frac{a \Delta z}{c^2}$$

$$\text{or } \Delta t_1 > \Delta t_0$$

Time interval for absorption

Time interval for emission



Flat geometry $\Rightarrow \Delta t_0 = \Delta t_1$!
 \times not correct

Conclusion Spacetime must have a mathematical structure that looks like Minkowski space but has a curved geometry (\Rightarrow gravity)

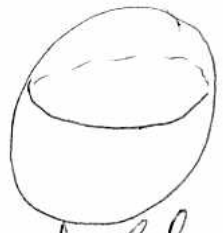
\Rightarrow manifold

Manifolds

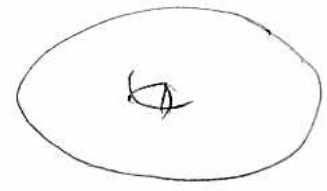
A n -dim set which locally "looks like" \mathbb{R}^n in continuity and smoothness.

Examples

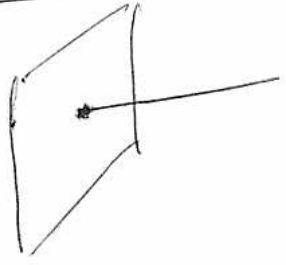
S^2



torus $S^1 \times S^1$



Not a manifold



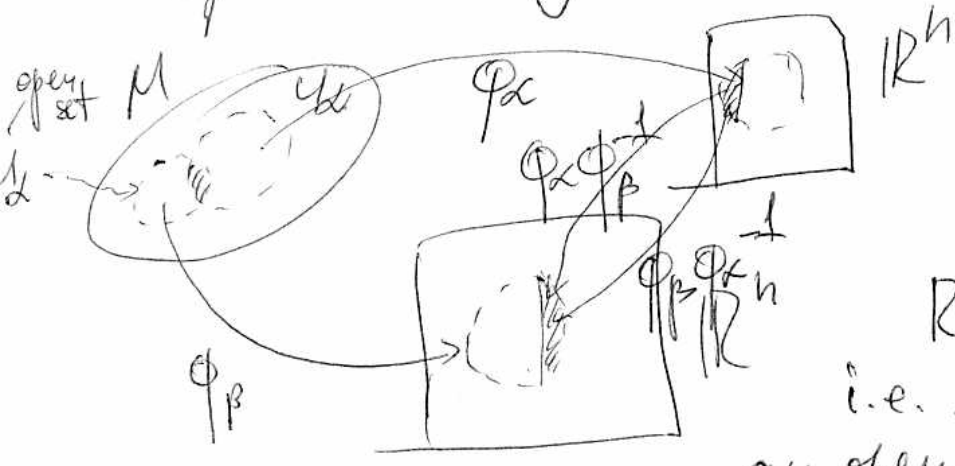
A function is C^p if its p th derivative exists and is continuous

C^∞ maps are smooth.

n -dim manifold M

Set obtainable by "smoothly sewing together"

open ball $|x-y| < R$.

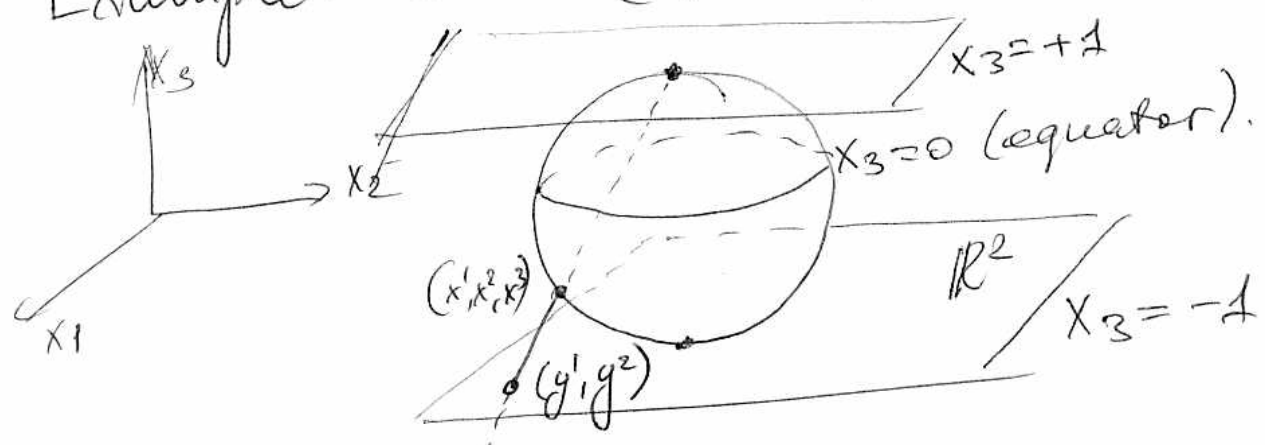


$\{(U_\alpha, \phi_\alpha)\}$ is a chart

Require each shaded region i.e. each $\phi_\alpha[U_\alpha \cap U_\beta]$ to be an open subset of \mathbb{R}^n and require $\phi_\alpha \circ \phi_\beta^{-1}$ is smooth (C^∞)

Two sets M and N are diffeomorphic if there exist a C^∞ map $\phi: M \rightarrow N$ with C^∞ inverse $\phi^{-1}: N \rightarrow M$; ϕ is a diffeomorphism

Example: S^2 $(x^1)^2 + (x^2)^2 + (x^3)^2 = 1$



$U_1 =$ sphere minus N Pole

$\phi_1(x^1, x^2, x^3) = (y^1, y^2) = \left(\frac{2x^1}{1-x^3}, \frac{2x^2}{1-x^3} \right)$ } (U_1, ϕ_1) form a chart

Can form another chart (U_2, ϕ_2)

$\phi_2(x^1, x^2, x^3) = (z^1, z^2) = \left(\frac{2x^1}{1+x^3}, \frac{2x^2}{1+x^3} \right)$

$U_2 =$ sphere minus S Pole

$\{(U_1, \phi_1), (U_2, \phi_2)\}$ atlas

Check that $\phi_2 \circ \phi_1^{-1}$ is given by $-1 < x^3 < 1$

$z^i = \frac{4y^i}{(y^1)^2 + (y^2)^2}$
 \hookrightarrow is C^∞