Phys 8501 Lecture 17 (T 11.02.2004)

Carroll Chap 9 #1

Assume $A_\mu J^\mu$ does not contribute to $T^{\mu\nu}$

2. Gravitational redshift

Consider an observer with 4-velocity u^μ but is stationary $u^i = 0$

$$u^\mu u_\mu = -1 \Rightarrow g_{\mu\nu} u^\mu u^\nu = -1$$

$$\Rightarrow g_{00} (u^0)^2 = -1$$

$$\Rightarrow u^0 = \sqrt{-\frac{1}{g_{00}}} = \left(1 - \frac{2GM}{r}\right)^{-1/2}$$

Along null geodesic

frequency $\omega = -p_\mu u^\mu$ wher $p_\mu = \frac{dx^\mu}{d\lambda}$

$$\omega = E\left(1 - \frac{2GM}{r}\right)^{-1/2} \quad = -\frac{dt}{d\lambda}\left(1 - \frac{2GM}{r}\right)^{+1/2}$$

For photon at r_1 and absorbed at r_2

$$\frac{\omega_2}{\omega_1} = \sqrt{\frac{1 - \frac{2GM}{r_1}}{1 - \frac{2GM}{r_2}}}$$

time translation $\leftarrow K^\mu \rightarrow (\partial_t)^\mu \quad E = -K_\mu \frac{dx^\mu}{d\lambda}$

Newtonian limit $r \gg 2GM$

$$\frac{\omega_2}{\omega_1} \overset{r \gg 2GM}{\simeq} 1 - \frac{GM}{r_1} + \frac{GM}{r_2} + \dots$$

$\longrightarrow$ gravitational redshift $= 1 + \Phi_1 - \Phi_2$

First detected by Pound & Rebka in 1960

Schwarzschild black holes

What happens at $r = 2GM$?

Coords (t, r, θ, ϕ)

Example : $ds^2 = -\frac{1}{t^4} dt^2 + dx^2$

Singularity at $t = 0$? $\quad -\infty < x < \infty$, $\; 0 < t < \infty$

Make a coord. transformation $t' = \frac{1}{t}$

$$\Rightarrow ds^2 = -dt'^2 + dx^2 \quad \text{flat spacetime}$$

In original coords (t, x) corresponds $t' > 0$.

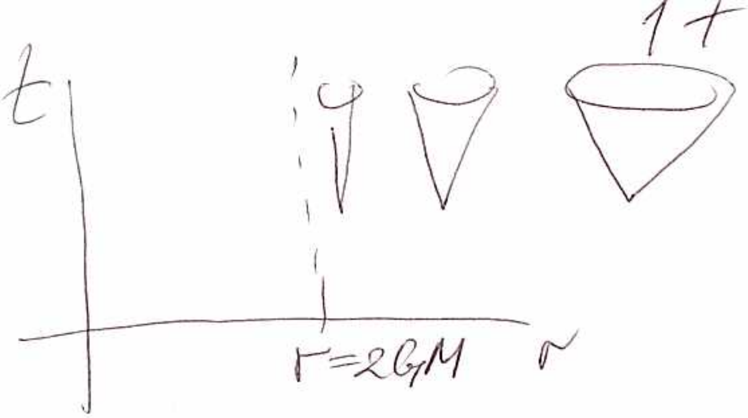
Consider the light cones of radial null curves $ds^2 = 0$ $\quad \theta = \phi = \text{const}$

$$ds^2 = 0 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2$$

$$\Rightarrow \frac{dt}{dr} = \pm \left(1 - \frac{2GM}{r}\right)^{-1}$$

$r \to \infty \quad \frac{dt}{dr} = \pm 1 \quad \checkmark$ o.k.

$r \to 2GM \quad dt/dr = \pm \infty \Rightarrow$ light near 2GM



An outside observer would perceive falling particle to travel more and more slowley.

Introduce new coords

Eddington-Finkelstein:

$r^* = r + 2GM \ln\left(\frac{r}{2GM} - 1\right)$

solution $\boxed{t = \pm r^* + \text{const}}$

$u = t - r^*$

$v = t + r^*$

Along null geodesic

$u = \text{const} = t - r^*$ $\quad t \to \infty \Rightarrow r^* \to \infty$ (outgoing)

$v = \text{const} = t + r^*$ $\quad t \to \infty \Rightarrow r^* \to -\infty$ (ingoing), $r \to 2GM$

$r \to 2GM \Rightarrow r^* \to -\infty$

$r \to \infty \Rightarrow r^* \to \infty$

Replace t with v

$ds^2 = \left(1 - \frac{2GM}{r}\right)dv^2 + (dr\,dv + dv\,dr) + r^2 d\Omega^2$

$r = 2GM \Rightarrow g_{vv} = 0.$

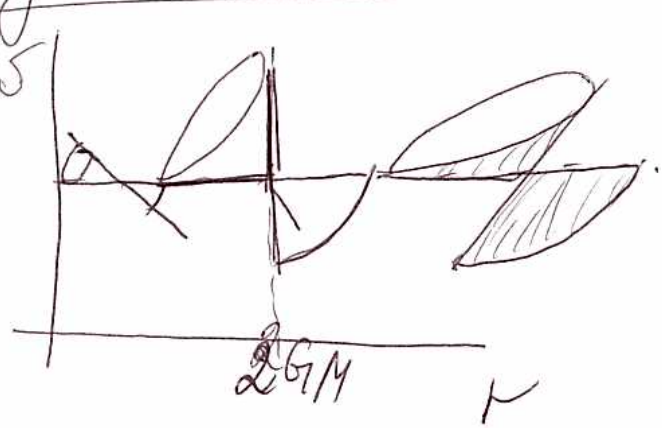
$\det g_{\mu\nu} = -r^4 \sin^2\theta$

radial null curves

$ds^2 = 0$ θ, φ const

$$\Rightarrow \frac{dv}{dr} = \begin{cases} 0 & \text{(ingoing)} \\ 2\left(1-\frac{2GM}{r}\right)^{-1} & \text{(outgoing)}. \end{cases}$$

<u>Light cones</u>



light cones tilt over for $r < 2GM$

Define <u>event horizon</u> to be a surface past wich particles can never escape to ∞

e.g. schwarzschild metric $R_\lambda = 2GM$.

A <u>black hole</u> is a region of spacetime separated from infinity by an event horizon

$\longrightarrow$ nothing can escape $\longrightarrow$ "black"

<u>Note</u> (1) Black holes don't suck in everything

(2) Newtonian escapability

$$\frac{1}{2} m v^2 = \frac{GmM}{r} \Rightarrow$$

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

$$v = c \Rightarrow$$

$$r = 2GM$$

$\Rightarrow$ Bh in Newtonian physics?

No! Because can have accelerated trajectories