

Phys 8501 Lecture 14 (M.10.25.2004)

Cosmological constant

In GR the absolute value of energy matters.

Define vacuum energy = energy of empty space.

$$\boxed{T_{\mu\nu}^{(vac)} = -\rho_{vac} g_{\mu\nu}}$$

— unique Lorentz invariant (0,2) tensor

Compare to perfect fluid:

$$T_{\mu\nu} = (\rho + p) u_{\mu} u_{\nu} + p g_{\mu\nu} = -\rho g_{\mu\nu}$$

true provided $\boxed{p_{vac} = -\rho_{vac}}$

Einstein equations become:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G (T_{\mu\nu}^{(M)} + T_{\mu\nu}^{(vac)}) = 8\pi G (T_{\mu\nu}^{(M)} - \rho_{vac} g_{\mu\nu})$$

Einstein introduced cosmological constant Λ

$$\Rightarrow \boxed{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}^{(M)}}$$

where $\rho_{vac} = \frac{\Lambda}{8\pi G}$

One can check that

$$S_{\Lambda} = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} (R - 2\Lambda) + \mathcal{L}_M \right]$$

leads to modified Einstein equations.

Remarkably cosmological observations imply

$$\left| \frac{\rho_{\Lambda}^{(\text{experiment})}}{\rho_{\Lambda}^{(\text{th})}} \right| \approx (10^{-12} \text{ GeV})^4 \approx (10^{-3} \text{ eV})^4$$

$$\frac{d\theta}{dt} = -4\pi(\rho + p_x + p_y + p_z)$$

what does theory predict?

Consider infinite number of harmonic oscillators \rightarrow each contributes $\frac{1}{2} \hbar \omega$ $\omega = \sqrt{k^2 + m^2}$

$$\text{but } \rho_{\text{vac}} \sim \sum_{k \leq k_{\text{max}}} \frac{\hbar \omega}{2} \sim \frac{1}{4} k_{\text{max}}^4$$

We expect GR to be valid up to $E \sim E_{\text{pl}} \sim 10^{18} \text{ GeV}$

$$\Rightarrow \rho_{\Lambda}^{(\text{th})} \sim (10^{18} \text{ GeV})^4$$

$$\left| \frac{\rho_{\Lambda}^{(\text{th})}}{\rho_{\Lambda}^{(\text{exp})}} \right| \sim \left(\frac{10^8}{10^{-12}} \right)^4 \sim 10^{120}$$

\rightarrow famous "cosmological constant problem."

Energy conditions

Impose constraints that limit the arbitrariness of $T_{\mu\nu}$

$$t^{\mu\nu} (R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}) = (8\pi G T_{\mu\nu}) + M^{\mu\nu}$$

Weak energy condition

WEC

$T_{\mu\nu} t^{\mu\nu} \geq 0$ for all timelike t^{μ} .

perfect fluid: $\rho \geq 0$ $\rho + p \geq 0$

Dominant Energy Condition DEC

$T_{\mu\nu} t^{\mu\nu} \geq 0$ and $T_{\mu\nu} t^{\nu}$ is nonspacelike
current J^{μ}
density

Strong Energy condition SEC

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$R_{\mu\nu} = 8\pi G (T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu})$$

$$(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu}) t^{\mu\nu} \geq 0.$$

Note (1) For $\rho \geq 0$ the energy conditions imply
 $w = \frac{p}{\rho} \geq -1$