Phys 8501 Lecture 14 (M.10.25.2004)

Cosmological constant

In GR the *absolute* value of energy matters.

Define vacuum energy = energy of empty space.

$$\boxed{T^{(vac)}_{\mu\nu} = -\rho_{vac} g_{\mu\nu}}$$ — *unique* Lorentz invariant (0,2) tensor

Compare to perfect fluid:

$$T_{\mu\nu} = (\rho + p) U_\mu U_\nu + p g_{\mu\nu} = -\rho g_{\mu\nu}$$

true provided $\boxed{p_{vac} = -\rho_{vac}}$

Einstein equations become:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G \left(T^{(M)}_{\mu\nu} + T^{(vac)}_{\mu\nu}\right) =$$

$$= 8\pi G \left(T^{(M)}_{\mu\nu} - \rho_{vac} g_{\mu\nu}\right)$$

Einstein introduced cosmological constant Λ

$$\Rightarrow \boxed{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T^{(M)}_{\mu\nu}}$$

where $\rho_{vac} = \frac{\Lambda}{8\pi G}$

One can check that

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$$S_\Lambda = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G_N} (R - 2\Lambda) + \mathcal{L}_M \right)$$

leads to modified Equation equations.

Remarkably cosmological observations imply

$$|\rho_\Lambda^{(\text{experiment})}| \simeq (10^{-12} GeV)^4 \simeq (10^{-3} eV)^4$$

$$\frac{d\theta}{d\tau} = -4\pi(\rho + p_x + p_y + p_z)$$

What does theory predict?

Consider infinite number of harmonic oscillators $\rightarrow$ each contributes

$$\frac{1}{2}\hbar\omega \qquad \omega = \sqrt{k^2+m^2}$$

But $\rho_{vac} \sim \sum^{k_{max}} \frac{\hbar\omega}{2} \sim \hbar k_{max}^4$

We expect GR to be valid up to $E \sim E_p \sim 10^{18} GeV$

$\Rightarrow \rho_\Lambda^{(th)} \sim (10^{18}$

$$\left|\frac{\rho_\Lambda^{(th)}}{\rho_\Lambda^{(exp)}}\right| \sim \left(\frac{10^8}{10^{-12}}\right)^4 \sim 10^{120}$$

$\longrightarrow$ famous " cosmological constant problem.

Energy conditions

Impose constraints that limit the arbitrariness of $T_{\mu\nu}$

$$t^\mu t^\nu \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) = \underbrace{\left(8\pi G T_{\mu\nu}\right) t^\mu t^\nu}_{\geq 0}$$

Weak energy condition WEC

$$T_{\mu\nu} t^\mu t^\nu \geq 0 \text{ for all timelike } t^\mu.$$

perfect fluid: $\rho \geq 0 \quad \rho + p \geq 0.$

Dominant Energy Condition DEC

$$T_{\mu\nu} t^\mu t^\nu \geq 0 \text{ and } \underbrace{T^{\mu\nu} t_\nu}_{\text{current density } J^\mu} \text{ is nonspacelike}$$

Strong Energy condition SEC

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$R_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right)$$

$$\left(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right) t^\mu t^\nu \geq 0.$$

Note (1) For $\rho \geq 0$ the energy conditions imply

$$\Rightarrow w = \frac{p}{\rho} \geq -1$$