Phys 8911 Lecture 6 (M 07.02.2005)

N=1 massive irreps

s \ Y	0	$\frac{1}{2}$	1	$\frac{3}{2}$
0	2	1		
$\frac{1}{2}$	1	2	1	
1		1	2	1
$\frac{3}{2}$			1	2
2				1

"chiral" "vector" multiplet

N=1 massless irreps

λ \ K	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$
2	1							1
$\frac{3}{2}$	1	1					1	1
1		1	1			1	1	
$\frac{1}{2}$			1	1	1	1		
0			1	1	1 1			
$-\frac{1}{2}$			1	1	1	1		
-1		1	1			1	1	
$-\frac{3}{2}$	1	1					1	1
-2	1							1
					4	4	4	4

Add CPT conjugates

← total # states $(4=2+2)$

$|\Omega\rangle = |m, k\rangle$ ← superhelicity

$k = 0, \pm\frac{1}{2}, \pm 1, \pm \ldots$

obtain 2 massless particles of helicity k, $k+\frac{1}{2}$

$\lambda, -\lambda \to$ spin $|\lambda|$.

Comments

1. Massless irreps are "half as big" as the massive irreps

$\hookrightarrow a_1^+, a_\alpha^+ \longrightarrow 4$

$$\{Q_\alpha \bar{Q}_{\dot\alpha}\} = 2m \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = -2m\,\sigma^0_{\alpha\dot\alpha}$$

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 4E \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

2. $\kappa = 0$ and $\kappa = \frac{1}{2}$ massless irreps $\Rightarrow$ same content as $Y = \frac{1}{2}$ massive irrep.

Extended supersymmetry

Can have more than one spinor generator Q_α^A where $A = 1, \ldots, N$

Obtain $[P^m, Q_\alpha{}^A] = 0 = [P^m, \bar{Q}_\alpha{}^A]$

$$[M_{ab}, Q_\alpha{}^A] = (\sigma_{ab})_\alpha{}^\beta Q^A_\beta$$

$$\{Q_\alpha^A \bar{Q}_{\dot\beta B}\} = 2\sigma^m_{\alpha\dot\beta} P_m \delta^A{}_B$$

However, $\{Q_\alpha{}^A Q_\beta{}^B\} = \varepsilon_{\alpha\beta} X^{AB}$, where $X^{AB} = -X^{BA}$

internal symmetry

Can show $[X^{AB}, Q_\alpha{}^C] = [X^{AB}, \bar{Q}_{\dot\alpha C}] = [X^{AB}, B^\lambda] = [X^{AB}, X^{CD}] = 0$

i.e. X^{AB} commutes with everything →
known as central charges.
see W & B Chap 1.

Note When $X^{AB}=0$ the 4d SUSY algebra
has a $U(N)$ sym

$$Q_\alpha^A \to U^A{}_B\, Q^B{}_\alpha\,, \quad \bar{Q}_{\dot\alpha A} \to \bar{Q}_{\dot\alpha B}\, U^{+B}{}_A$$

where $U^A{}_B$ is a unitary matrix $N \times N$.
When $X^{AB} \neq 0$ this sym is reduced
e.g. $N=1$, $X^{AB}=0$ and the $U(1)$ symmetry
is the R-symmetry.

$N>1$ representations of SUSY

1. Massless states with no central charges
There are N creation operators

$$a_A^+ \quad (A=1,\dots,N)$$

satisfying

$$\{a_A a_B^+\} = \delta_{AB} \qquad \{a_A a_B\} = 0 = \{a_A^+, a_B^+\}$$

Start with the ground state $|\Omega\rangle$ with
superhelicity κ can form

$$\frac{1}{\sqrt{n!}}\, a_1^+ \dots a_n^+ |\Omega\rangle \qquad 1 \le n \le N$$

This gives

helicity (λ)	k	$k+\frac{1}{2}$	$k+1$	$\ldots$	$k+\frac{N}{2}$
# of states	$1 = \binom{N}{0}$	$N = \binom{N}{1}$	$\binom{N}{2}$		$\binom{N}{N} = 1$

Total # of States

$$= \sum_{k=0}^{N} \binom{N}{k} = 2^N$$

Note this is not a CPT eigenstate unless

$$-k = k + \frac{N}{2} \Rightarrow \boxed{k = -\frac{N}{4}}$$

So, in general need to include

$-k, -k-\frac{1}{2}, \ldots, -k-\frac{N}{2}$ helicity states

Examples: $N=2$

$k=0$

λ	0	$\frac{1}{2}$	1	CPT conj $\rightarrow$	0	$-\frac{1}{2}$	-1
# states	1	2	1		1	2	1

combine

λ	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
# states	1	2	2	2	1

~~This~~ $N=2$ vector multiplet.

$k = -\frac{1}{2}$

λ	$-\frac{1}{2}$	0	$\frac{1}{2}$
# states	1	2	1

$N=2$ hypermultiplet automatically CPT complete

Fayet!

<u>$N=4$</u> $\quad k=-1$

λ	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$+1$
#states	1	4	6	4	1

$N=4$ vector multiplet
auto CPT complete

<u>Note</u> 1. Require renormalizability excludes $|\lambda|>1$
$\Rightarrow N=4$ max possible (global) supersymmetry.

~~$N=4$ vector multiplet~~
~~auto CPT complete~~

2. If exclude $|\lambda|>2$ then max. amount of SUSY $N=8$ (local) supersymmetry $\rightarrow$ supergravity.

2. <u>Massive states - no central charges</u>

There are now $2N$ creation ops $(a_\alpha^A)^+$
$A=1\ldots N$
$\alpha=1,2$

satisfying $\{a_\alpha^A\, a_\beta^{B+}\}=\delta_{\alpha\beta}\,\delta^{AB}$

There are $2^{2N}(2Y+1)$ states in each massive irrep.

For each "N" superspin $Y_{(N)}$ obtain "$N-1$" superspin

$$\begin{cases} Y_{(N-1)}-\frac{1}{2},\ Y_{(N-1)},\ Y_{(N-1)}+\frac{1}{2} & Y\neq 0 \\ 0,\ 0,\ \frac{1}{2} & Y=0 \end{cases}$$

where $Y_{(0)}=s=$ spin of particle

<u>Example</u>: $N=2 \quad Y_{(2)}=0$

$0,\ 0,\ \frac{1}{2}$ $\quad Y_{(1)}$ superspins

$5\times 0,\ 4\times\frac{1}{2},\ 1\times 1$ spin states $\Longleftarrow$ spin $Y_{(0)}$ $\quad 0,0,\frac{1}{2} \qquad 0,0,\frac{1}{2} \qquad 0,\frac{1}{2},\frac{1}{2},1$