

Phys 8911 (18.01.2005)  
Introduction to Supersymmetry

Texts

"Supersymmetry and Supergravity" Wess & Bagger

"Supersymmetric gauge field theory & String theory" Bailin & Love

Also P. West "Intro to Susy & Supergravity"  
S. Weinberg "QFT Vol III"  
Buchbinder & Kuzenko "Ideas and Methods of Supersymmetry & Supergr."

Review articles

S.P. Martin hep-ph/9709356  
[www.arxiv.org](http://www.arxiv.org)

J. Lykken hep-th/9612114

M. Sohnius Phys. Rept. 128, 39 (1985)

Outline

1. Formal Developments
2. Applied Developments

1. → Motivation, Coleman-Mandula thm,  
4d Susy algebra and representation  
Superspace and superfields  
Susy gauge theories, Nonrenormalisation thm,  
Susy standard model.

# Supersymmetry breaking Phenomenology / Supergravity.

Motivation Why study supersymmetry?

① Supersymmetry is the only possible extension of the known spacetime symmetries.

↳ energy-momentum  $p_\mu$   
Lorentz transf.  $M_{\mu\nu}$

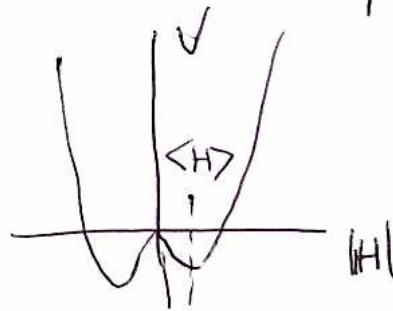
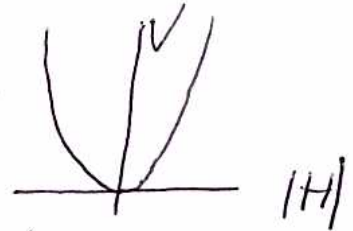
② Hierarchy problem

Higgs boson

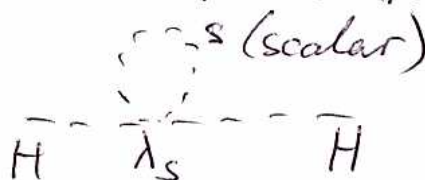
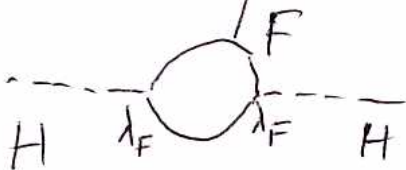
Higgs scalar potential  $V = m_H^2 |H|^2 + \lambda |H|^4$

If  $m_H^2 < 0$  then min. occurs

$$\text{at } \langle H \rangle = \sqrt{\frac{-m_H^2}{2\lambda}}$$



experimentally  $\langle H \rangle = 174 \text{ GeV} \Rightarrow -m_H^2 \sim (100 \text{ GeV})^2$   
Consider quantum corrections to  $m_H^2$ :





$$\Delta m_H^2 \sim \frac{1}{16\pi^2} (\lambda_S - \lambda_F^2) \Lambda_{UV}^2$$

But  $\Lambda_{UV} \sim \underbrace{M_p}_{\text{Planck scale}} \sim 10^{19} \text{ GeV}$

$$\lambda_S \sim \lambda_F \sim 1 \Rightarrow \Delta m_H^2 \gg (100 \text{ GeV})^2$$

Supersymmetry  $\Rightarrow \lambda_S = \lambda_F^2$

$\Rightarrow \Delta m_H^2$  not quadratically sensitive to  $\Lambda_{UV}$

Solves hierarchy problem!

Review spacetime symmetries

Let  $x^m$  ( $m=0,1,2,3$ ) denote a spacetime point

$$x'^m = \Lambda^m_n x^n + \epsilon^m$$

Leaves the metric  $ds^2 = \eta_{mn} dx^m dx^n$  invariant where  $x^m = (x^0, x^1, x^2, x^3)$   $x^0 = ct$   $\eta_{mn} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$

$$\det \Lambda = +1 \text{ and } \Lambda^0_0 \geq 1$$

The transformations  $(\Lambda, \epsilon)$  are known as Poincaré transformations

When  $b=0$ ;  $\Lambda$  are the Lorentz transformations satisfying  $\Lambda^T \eta \Lambda = \eta$

The union of all {Poincaré} transformations  
{Lorentz} are known as {Poincaré} group denoted by  
{Lorentz} (proper, achronous)

$$\left\{ \begin{array}{l} \Pi \\ SO(3,1)^\uparrow \end{array} \right\}$$

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Lorentz group

1 Ref. Bailin & Love  
Wess & Bagger  
Buchbinder & Kuzenko

$$SO(3,1)^\uparrow \cong SU(2) \times SU(2)$$

generators  $\frac{1}{2}(J_i \pm K_i)$   $i=1, 2, 3$   
 $J_i$  = rotation gen  
 $K_i$  = boost gen.

Representations are labelled by integers  $(m, n)$  with spin  $m+n$  where  $m, n = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots$

- Egs
- $(0, 0)$  spin 0, scalar rep.
  - $(\frac{1}{2}, 0)$  spin  $\frac{1}{2}$ , "left handed" spinor
  - $(0, \frac{1}{2})$  spin  $\frac{1}{2}$ , "right handed" spinor
  - $(\frac{1}{2}, 0) \otimes (0, \frac{1}{2}) = (\frac{1}{2}, \frac{1}{2})$  spin 1, ~~rep.~~ vector rep.  $\rightarrow$  4-vector  $V^\mu$

$$\left(\frac{1}{2}, \frac{1}{2}\right) \otimes \left(\frac{1}{2}, \frac{1}{2}\right) \supset (1, 1)_S$$

- spin 2 → symmetric tensor type  
symmetric

see Ramond "Field Theory"

metric perturbation.

## Lorentz algebra

Simply connected groups have a 1-1 correspondence between rep of group and algebra  
 $SO(3,1)$  is not simply connected.

the "universal covering group" (that will be simply connected).

→  $SL(2, \mathbb{C})$  which is simply connected

↳ set of  $2 \times 2$  complex unimodular matrices  
 (det = 1)

$$x' = \Lambda x.$$