Chain Maps and Homotopies

$$f: X_\cdot \to Y_\cdot \quad \boxed{\circlearrowleft} \quad \Rightarrow \quad \cancel{H(f)} \; f_* = H_\cdot(f): H_\cdot(X) \to H_\cdot(Y)$$

$$s: X_\cdot \to Y_{\cdot+1} \qquad ds + sd = f - g$$

same maps of Homology

Tensor Product

$$(X_\cdot \otimes Y_\cdot)_n = \bigoplus_{i+j=n} X_i \otimes Y_j$$

$$d(x \otimes y) = dx \otimes y + (-1)^{|x|} x \otimes dy$$

$$d_{X \otimes Y} = d \otimes id + id \otimes d$$

$I_\cdot \qquad 0 \to R \to R \oplus R \to 0$

$[I] \quad [0] \; [1]$

$$d[I] = [1] - [0]$$

Then a homotopy is $X_\cdot \otimes I_\cdot \xrightarrow{h} Y_\cdot$ s.t. $s[1] = f$, $s[0] = g$ (betw. f and g)

$$h(x_j \otimes [1]) = f(x)$$
$$h(x_j \otimes [0]) = g(x)$$
$$h(x \otimes [I]) = (-1)^i s(x)$$

~~Propert~~ Consequences of Axioms

$E_q(X, A)$ generalized h theory (when dimension axiom is dropped)

Existence any $H_q(X, A; G)$

Uniqueness if you have $E_\cdot(X, A) \to E'_\cdot(X, A)$ inducing iso on $(X, A) = (*, \emptyset)$

Reduced, based vs unbased

$\tilde{E}_q(X) = E_q(X, *)$, if X is based CW cx

X retracts on $* \Rightarrow E_\cdot(X) \cong \tilde{E}_\cdot(X) \oplus E_\cdot(*)$ and $\tilde{E}_\cdot(X) = \mathrm{Ker}[E_q(X) \to E_q(*)]$

because $\to E_q(*) \rightleftarrows E_q(X) \to \tilde{E}_q(X) \to E_{q-1}(*)$

$* \in A \subset X \quad E_\cdot(*)$ maps isomorphically under $E_\cdot(A) \to E_\cdot(X)$

$\Rightarrow$ LES $\quad \ldots \to \tilde{E}_q(A) \to \tilde{E}_q(X) \to E_q(X, A) \xrightarrow{\partial} \tilde{E}_{q-1}(A) \to$