Math 8306 Lecture 2 (09.10.04)

<u>CW complexes</u> → closure fineteness
→ weak topology

built out of discs:

$D^n = \{x \in \mathbb{R}^n \mid |x| \le 1\}$ n-disk

$\partial D^n = S^{n-1}$

union

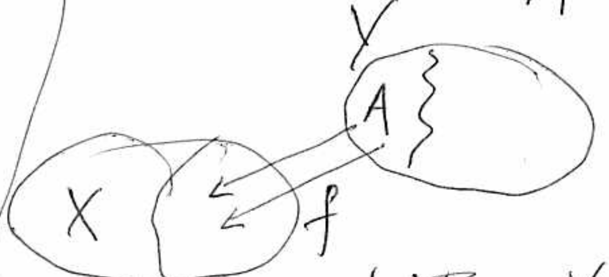
Def. A <u>CW complex</u> is a space $X = \bigcup_{n=0}^{\infty} X^n$
s.t. $X^0 =$ discrete space

$X^{n+1} = X^n \cup_j \left(\underbrace{\coprod_\alpha D^{n+1}_\alpha}_{\text{disjoint union}} \right)$

glue ⇝ topological union (pushout)

<u>Pushout (topological union)</u>:

$X \cup_f Y$, $f: A \to X$, $A \subset Y$



$X \cup_f Y = X \amalg Y \Big/ \left\{ \begin{array}{l} a \sim f(a) \\ \forall a \in A \end{array} \right\}$

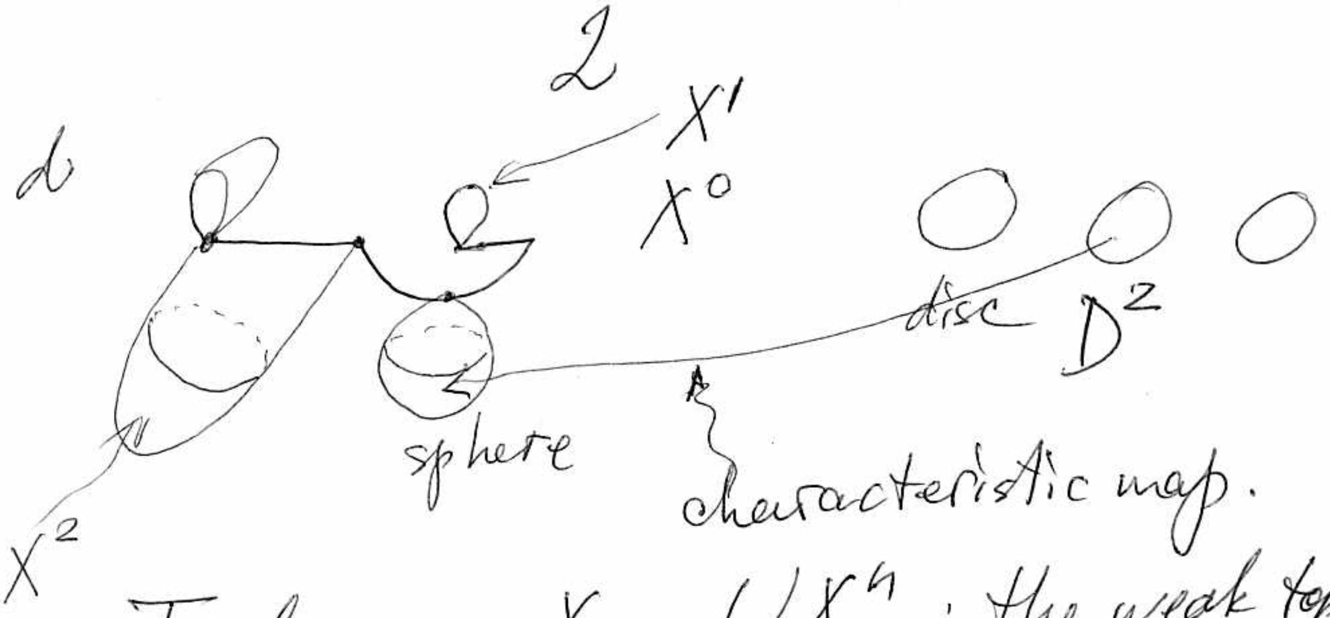
properties: $X \subset X \cup_f Y \xleftarrow[\text{map exists}]{} Y$

X is subsp. of the pushout

equotion topology?

defined by a given map $j: \coprod_\alpha S^n_\alpha \to X^n$

gluing map



Topology on $X = \bigcup_{n \geq 0} X^n$; the weak topology or the topology of the union:

a set in X is closed iff its intersection with each X^n is closed in X^n

if and only if.

The images of the disks are called cells.

A <u>cell</u> is the image of a disk D_α^n.

$D_\alpha^n \to X$ is the characteristic map (of the cell)

$X^n = n$ - skeleton $\subset X$ closed subspace

<u>Def</u> A <u>subcomplex</u> $A \subset X$ (X CW complex) is a subspace and a CW complex, s.t. each cell $D^n \to \cancel{A} \subset X$ is a cell of X

(or $A = \bigcup$ some cells of X).

<u>Def</u> A <u>cellular map</u> $f: X \to Y$ is a conts map taking X^n to Y^n $\forall n$

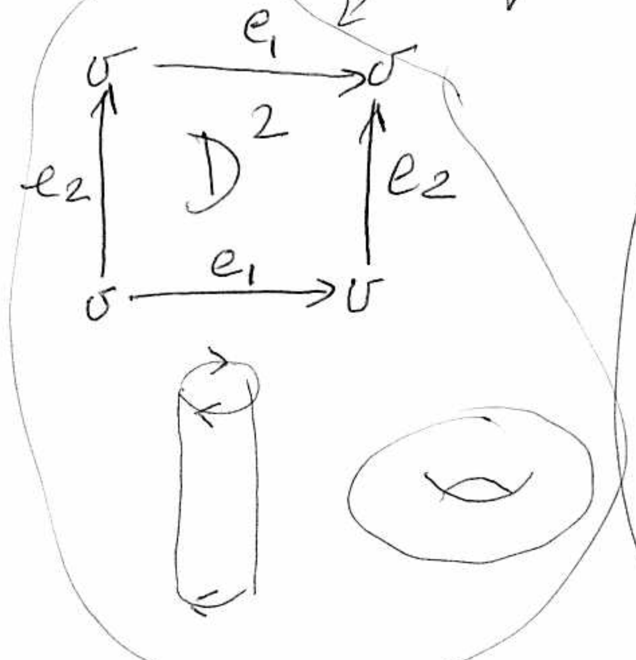
<u>Examples</u> 1. A graph is 1-dim CW complex.

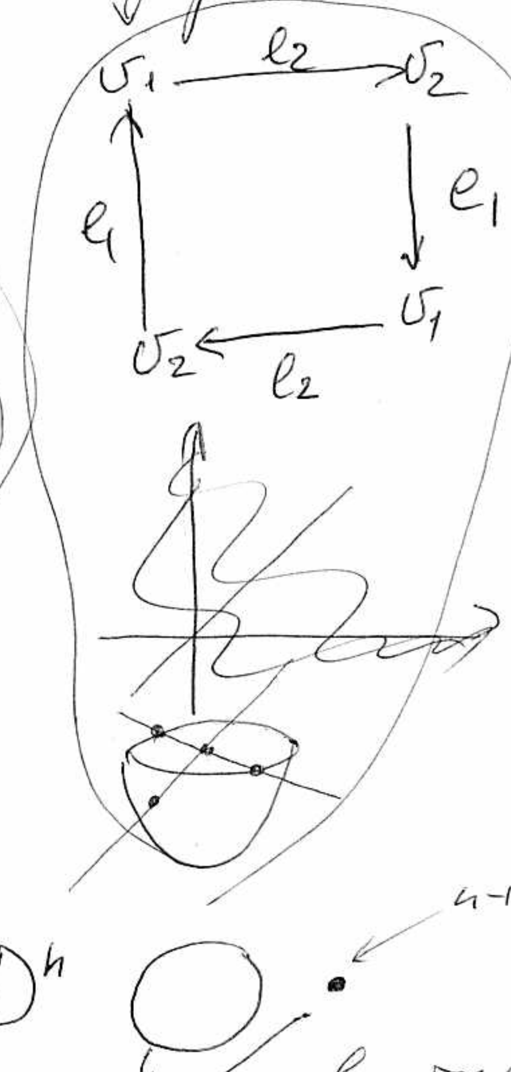
2. $T^2 = S^1 \times S^1$, $\mathbb{R}P^2$, K

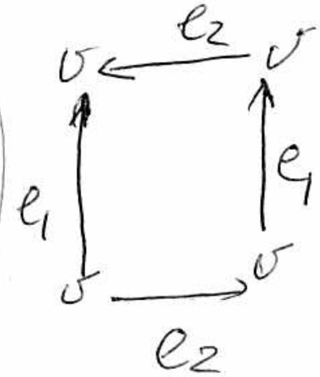
Top

real projective plane

klein bottle







3. S^n D^n

n-1 skeleton

все точки границы идентифицируются в одну

2

cellular map:

$S^m \longrightarrow S^n$

for $m<n$ $\Longrightarrow$ constant.

$S^m \to S^n$ for $m \geqslant n$ a based map

$\Longrightarrow$ cellular

if you map it with rotation $\to$ not a cellular map.

character. map is not nessecerely cellular.