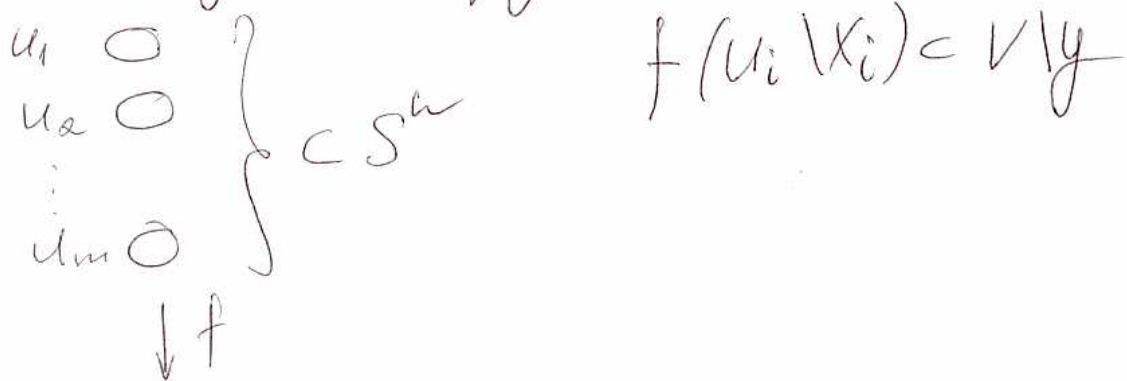


Local Degree

$f: S^n \rightarrow S^n, n > 0$

Suppose for some point $y \in S^n$, its preimage $f^{-1}(y)$ is finite = $\{x_1, \dots, x_m\}$

Choose neighborhoods U_1, \dots, U_m of these points and neighborhood V of y , s.t. $f: U_i \rightarrow V$



Choose any i :
 $V \circ \subset S^n$

$H_n(S^n, S^n \setminus \{x_i\})$

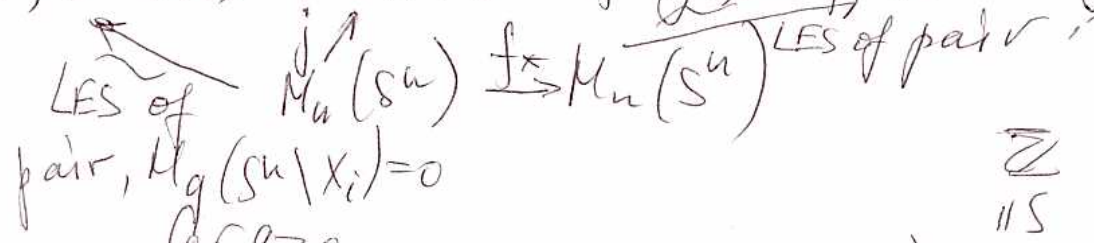


$M_n(x_i, B) \cong M_n(A, A \cap B)$

$x = A \cup B$
 open
 exist

$H_n(U_i, U_i \setminus x_i) \xrightarrow{f_*} M_n(V, V \setminus y)$

$H_n(S^n, S^n \setminus \{x_i\}) \xrightarrow{f_*} H_n(S^n, S^n \setminus \{y\})$



Outer vertical arrows: $H_n(U_i, U_i \setminus x_i) = H_n(S^n) = H_n(V, V \setminus y)$
 Then the upper f_* is multiplied by an integer, called local degree at x_i : $\deg f|_{x_i}$

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Claim: $\deg f = \sum_{i=1}^m \deg f|_{X_i}$

Remarks: 1. If f is a homeomorphism, then all maps in diagram are iso and $\deg f = \pm 1$ and $\deg f|_{X_i} = \pm 1$

2. If $f: U_i \rightarrow V$ is a homeomorphism, then again $\deg f|_{X_i} = \pm 1$

Proof: $H_n(S^u, S^u | f^{-1}(y)) \cong \bigoplus_{i=1}^m H_n(U_i, U_i | X_i)$
 $A = \coprod U_i, B = S^u \setminus f^{-1}(y) = S^u \setminus \{x_1, \dots, x_m\}$
 $H_n(\coprod U_i, \coprod U_i | X_i)$

Comm. of upper left $\Delta \Rightarrow p_i$ followed by excision

Choose $H_n(S^u) \cong \mathbb{Z}$

$$\text{lower } \Delta \Rightarrow p_i j(1) = 1 \Rightarrow j(1) = (1, 1, \dots, 1) = \sum_{i=1}^m k_i(1)$$

upper $\square \Rightarrow$

$$f_* h_i(1) = \deg f|_{X_i}$$

$$\sum f_* k_i(1) = \sum_{i=1}^m \deg f|_{X_i}$$

$$\parallel$$

$$f_* j(1)$$

lower $\square \Rightarrow f_* j(1) = \deg f$

□